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Research Article

Interaction of Two-Level Atom with Squeezed Vacuum Reservoir

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In this paper, the quantum properties of a two-level atom interaction with squeezed vacuum reservoir is throughly analyzed. With the aid of the interaction Hamiltonian and the master equation, we obtain the time evolution of the expectation values of the atomic operators. Employing the steady-state solution of these equations, we calculate the power spectrum and the second-order correlation function for the interaction of two-level atom with squeezed vacuum reservoir. It is found that the half width of the power spectrum of the light increases with the squeeze parameter, r. Furthermore, in the absence of decay constant and interaction time, it enhances the probability for the atom to be in the upper level.

1. Introduction

The interaction of squeezed light and atoms is a problem of fundamental interest in quantum optics [1, 2]. In addition, the interaction of two-level atoms with squeezed light has been extensively studied as it exhibits interesting nonclassical features. This kind of interaction is involved in various physical processes of interest such as resonance fluorescence and laser dynamics. Moreover, the effect of the squeezed light on the properties of the fluorescent light emitted by the two-level atom has attracted a great deal of interest [1–13]. In this respect, a two-level atom embedded in a broadband squeezed vacuum has studied by Gardiner [3] and found that the atomic polarisation quadratures are phase dependent. This inhibited phase decay results in a feature with subnatural linewidth in the fluorescent spectrum that is a direct indication of the noise reduction in one quadrature of the squeezed light field. In addition, the fluorescent spectrum when the atom immersed in a broadband squeezed vacuum and driven by coherent light have studied by Carmichael et al. [4].

The properties of the fluorescent light emitted by a two-level atom in a cavity driven by coherent light and coupled to a squeezed vacuum has been studied by several authors [3–14]. Some of these studies have shown that the width of the incoherent spectrum of the fluorescent light has been modified by the squeezed light. Furthermore, the composite system consisting of a two-level atom and a parametric amplifier has been considered by different authors. Among these authors, Tesfa [15] has studied the quantum properties of a single two-level atom with a single-mode squeezed radiation. In addition, Abebe and Gemechu [16] have studied the quantum properties of cavity-mode light and fluorescence light for a two-level atom inside a degenerate parametric oscillator, when the cavity is coupled to a vacuum reservoir. In this study, they found that the half-width of the power spectrum for the fluorescent light in the presence of a parametric amplifier increases, while it decreases for the cavity-mode light.

Moreover, Jin and Xiao [17] showed that the presence of the two-level atoms increases the degree of squeezing of the light produced by parametric oscillator considerably. Moreover, Alebachew [18] has studied a coherently driven two-level atom inside a parametric oscillator operating below the threshold. He found that the presence of the parametric amplifier leads to an increase in the width of the power spectrum of the fluorescent light in the weak and strong driving light limits and the effect of the presence of the parametric amplifier on the second-order correlation function is to enhance its decay rate. In addition, Tanas et al. [19] studied the effect of a finite-bandwidth squeezed vacuum on resonance fluorescence from a driven two-level atom. They have derived, applying the dressed-atom model, the master equation for the system which is valid for an arbitrary value of the Rabi frequency of the driving field. They have also investigated the squeezing induced changes to the spectral linewidths and intensities by analysing the quadrature-noise spectrum of the fluuorescence field.

In this paper, we analyze the quantum properties of the atomic variables when a two-level atom interacts with a squeezed vacuum reservoir. The system we have considered may be studied analytically using the master equation derived in the bad-cavity limit. However, the master equation for a two-level atom embedded in a squeezed vacuum is obtained, employing the bad-cavity limit, only contains the atomic density operator and will not be useful to study the cavity mode properties. It is then quite interesting to study the effect of squeezed light on the quantum properties of the atomic variables of the two-level atom. We derive the equations of evolution for the expectation values of atomic operators using the master equation in the bad-cavity limit. Applying the resulting equations, we calculate the power spectrum and the second-order correlation function of the atomic variables when a two-level atom interacts with a squeezed vacuum reservoir.

2. The Master Equation

For pedagogical reasons we derive here the master equation for a system (a two-level atom) coupled to a squeezed reservoir. Various derivations of this master equation can be found in the literature including quantum-optics textbooks and recent regular articles [20–29]. Here, the master equation has been derived for the reduced atomic density matrix under the Born-Markov approximation which takes into account the dependence of the relaxation rates on the strength of the laser field. We now seek to obtain the equation of evolution of the reduced density operator for a two-level atom embedded in a squeezed vacuum. A system coupled to a reservoir can be described by the Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_{SR}. \tag{1}$$

The equation of evolution of density operator is given by

$$\frac{d}{dt}\hat{X}(t) = -i\left[\hat{H}_S(t) + \hat{H}_{SR}, \hat{X}(t)\right], \tag{2}$$

where $\hat{X}(t)$ is the density operator for the system. Using equation (2), the reduced density operator $\hat{\rho}(t) = Tr_R\hat{X}(t)$ evolves in time according to

$$\frac{d}{dt}\hat{\rho}(t) = -i\left[\hat{H}(t), \hat{\rho}(t)\right] - iTr_R\left[\hat{H}_{SR}(t), \hat{X}(t)\right], \tag{3}$$

in which Tr_R indicates the trace over the reservoirs variables only. On the other hand, a formal solution of Eq. (2) can be written as

$$\hat{X}(t) = \hat{X}(0) - i\int_0^t \left[\hat{H}_S\left(t'\right) + \hat{H}_{SR}\left(t'\right), \hat{X}\left(t'\right)\right]dt'. \tag{4}$$

In order to obtain mathematically manageable that $\hat{X}(t')$ by some approximately valid expression, then, in the first place, we would arrange the reservoir in such a way that its density operator $\hat{R}$ remains constant in time. This can be achieved by letting a beam of light in a squeezed vacuum state of constant intensity fall continuously on the system. Moreover, we decouple the system and reservoirs density operators, so that

$$\hat{X}\left(t'\right) = \hat{\rho}\left(t'\right)\hat{R}. \tag{5}$$

With the aid of this, one can rewrite Eq. (4) as

$$\hat{X}\left(t'\right) = \hat{\rho}(0)\hat{R} - \int_0^t \left[\hat{H}_S\left(t'\right) + \hat{H}_{SR}\left(t'\right), \hat{\rho}\left(t'\right)\hat{R}\right]dt'. \tag{6}$$

Now on substituting (6) into (3), there follows

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) = {}& -i\left[\hat{H}_{SR}(t), \hat{\rho}(t)\right] - i\left[\left\langle\hat{H}_{SR}(t)\right\rangle_R, \hat{\rho}(0)\right] \\ & - \int_0^t \left[\left\langle\hat{\rho}_{SR}(t)\right\rangle_R, \left[\hat{H}_S\left(t'\right), \hat{\rho}\left(t'\right)\right]\right]dt' \\ & - \int_0^t Tr_R\left[\hat{H}_{SR}(t), \left[\hat{H}_{SR}\left(t'\right), \hat{\rho}\left(t'\right)\hat{R}\right]\right]dt',\end{aligned} \tag{7}$$

where the subscript R indicates that the expectation value is to be calculated using the reservoirs density operator $\hat{R}$.

As we know, a light mode confined in a cavity, usually formed by two mirrors, is called a cavity mode. A commonly used cavity has a single-port mirror. One side of such a cavity is a mirror through which light can enter or leave the cavity, with the other side being a mirror through which light may enter but cannot leave the cavity. We now proceed to obtain the equation of evolution of the reduced density operator, in short the master equation, for a cavity mode coupled to a squeezed vacuum reservoir via a single-port mirror. We consider the reservoir to be composed of a large number of submodes. Thus, according to [20] with $\hbar = 1$, the interaction between a two-level atom and a squeezed vacuum reservoir can be described by the Hamiltonian

$$\hat{H}_{SR} = i\sum_k g_k\left[\hat{\sigma}_+\hat{b}_k e^{i(\omega_0-\omega_k)t} - \hat{b}_k^{\dagger}\hat{\sigma}_- e^{-i(\omega_0-\omega_k)t}\right], \tag{8}$$

where $\hat{b}_k$ is the annihilation operator for a reservoir submode characterized by the wave vector $\vec{k}$ and g_k is the coupling constant. For this Hamiltonian, it is easy to see that

$$[\langle \hat{H}_{SR}(t)\rangle_R, \hat{\rho}(0)] = [\langle \hat{H}_{SR}(t)\rangle_R, [\langle \hat{H}_S(t')\rangle, \hat{\rho}(t')]] = 0. \quad (9)$$

In view of these results, expression (7) reduces to

$$\begin{aligned}\frac{d}{dt}\hat{\rho} = &-i[\hat{H}_S(t), \hat{\rho}] - \int_0^t Tr[\hat{R}\hat{H}_{SR}(t)\hat{H}_{SR}(t')\hat{\rho}(t')]dt' \\ &- \int_0^t \hat{\rho}(t') Tr[\hat{R}\hat{H}_{SR}(t')\hat{H}_{SR}(t)]dt' \\ &+ \int_0^t Tr[\hat{H}_{SR}(t')\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t')]dt' \\ &+ \int_0^t Tr[\hat{H}_{SR}(t')\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t)]dt'.\end{aligned} \quad (10)$$

Furthermore, using Hamiltonian described by (8) we have

$$\begin{aligned}&-Tr_R(\hat{H}_{SR}(t')\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t)) \\ &= -[\Gamma_1\hat{\sigma}_+\hat{\rho}(t')\hat{\sigma}_- + \Gamma_2\hat{\sigma}_-\hat{\rho}(t')\hat{\sigma}_+ \\ &\quad + \Gamma_3\hat{\sigma}_-\hat{\rho}(t')\hat{\sigma}_- + \Gamma_4\hat{\sigma}_+\hat{\rho}(t')\hat{\sigma}_+],\end{aligned} \quad (11)$$

where

$$\Gamma_1 = -\sum_{k,k'} g_k g_{k'} \langle \hat{b}_k^\dagger \hat{b}_{k'}\rangle_R \exp[-i(\omega_0-\omega_k)t + i(\omega_0-\omega_{k'})t'], \quad (12)$$

$$\Gamma_2 = -\sum_{k,k'} g_k g_{k'} \langle \hat{b}_k \hat{b}_{k'}^\dagger\rangle_R \exp[i(\omega_0-\omega_k)t + i(\omega_0-\omega_{k'})t'], \quad (13)$$

$$\Gamma_3 = \sum_{k,k'} g_k g_k{}' \lambda_k \langle \hat{b}_k^\dagger \hat{b}_{k'}^\dagger\rangle_R \exp[-i(\omega_0-\omega_k)t - i(\omega_0-\omega_{k'})t'], \quad (14)$$

$$\Gamma_4 = \sum_{k,k'} g_k g_k{}' \langle \hat{b}_k \hat{b}_{k'}\rangle_R \exp[-i(\omega_0-\omega_k)t + i(\omega_0-\omega_{k'})t']. \quad (15)$$

Now applying the relation described by [20]

$$\langle \hat{b}_k^\dagger \hat{b}_{k'}\rangle_R = N\delta_{kk}{}', \quad (16)$$

$$\langle \hat{b}_k \hat{b}_{k'}^\dagger\rangle_R = (N+1)\delta_{kk}{}', \quad (17)$$

$$\langle \hat{b}_k \hat{b}_{k'}\rangle_R = \langle \hat{b}_k^\dagger \hat{b}_{k'}^\dagger\rangle_R = -M\delta_{k,2k_{0-k}}, \quad (18)$$

where $N = \sinh^2 r$, $M = \cosh r \sinh r$, and the squeeze parameter r is taken for convenience to be real and positive. In view of these results, we arrive at

$$\Gamma_1 = -\gamma N\delta(t-t'), \quad (19)$$

$$\Gamma_2 = -\gamma(N+1)\delta(t-t'), \quad (20)$$

$$\Gamma_3 = \Gamma_4 = -\gamma M\delta(t-t'). \quad (21)$$

in which γ is the atomic decay rate. Now, with the aid of (11) along with (19), (20), and (21), one readily finds

$$\begin{aligned}&\int_0^t Tr_R(\hat{H}_{SR}(t')\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t))dt' \\ &= \frac{\gamma}{2}[(N+1)\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ + N\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- + M(\hat{\sigma}_-\hat{\rho}\hat{\sigma}_- + \hat{\sigma}_+\hat{\rho}\hat{\sigma}_+)].\end{aligned} \quad (22)$$

It is also easy to note that

$$\begin{aligned}&\int_0^t Tr_R(\hat{H}_{SR}(t)\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t'))dt' \\ &= \frac{\gamma}{2}[(N+1)\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ + N\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- + M(\hat{\sigma}_-\hat{\rho}\hat{\sigma}_- + \hat{\sigma}_+\hat{\rho}\hat{\sigma}_+)].\end{aligned} \quad (23)$$

Moreover, using (10) along with the fact that

$$\hat{\sigma}_-^2 = \hat{\sigma}_+^2 = 0, \quad (24)$$

we readily arrive at

$$-Tr_R(\hat{R}\hat{H}_{SR}(t)\hat{H}_{SR}(t')) = \Gamma_1\hat{\sigma}_-\hat{\sigma}_+ + \Gamma_2\hat{\sigma}_+\hat{\sigma}_-, \quad (25)$$

where Γ_1 and Γ_2 are defined by (19) and (20), respectively. Thus on account of the results, we see that

$$\begin{aligned}&\int_0^t Tr_R(\hat{R}\hat{H}_{SR}(t)\hat{H}_{SR}(t'))\hat{\rho}(t')dt' \\ &= \frac{\gamma}{2}[(N+1)\hat{\sigma}_+\hat{\sigma}_-\hat{\rho} + N\hat{\sigma}_-\hat{\sigma}_+\hat{\rho}],\end{aligned} \quad (26)$$

$$\begin{aligned}&\int_0^t \hat{\rho}(t') Tr_R(\hat{R}\hat{H}_{SR}(t)\hat{H}_{SR}(t'))dt' \\ &= \frac{\gamma}{2}[(N+1)\hat{\rho}\hat{\sigma}_+\hat{\sigma}_- + N\hat{\rho}\hat{\sigma}_-\hat{\sigma}_+].\end{aligned} \quad (27)$$

Finally, on substituting (22), (23), (26), and (27) into (10), there follows

$$\begin{aligned}\frac{d}{dt}\hat{\rho} = &-i\left[\hat{H}_S(t), \hat{\rho}\right] \\ &+ \frac{\gamma}{2}(N+1)\left[2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\right] \\ &+ \frac{\gamma}{2}N\left[2\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho} - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\right] \\ &+ \gamma M\left[\hat{\sigma}_-\hat{\rho}\hat{\sigma}_- + \hat{\sigma}_+\hat{\rho}\hat{\sigma}_+\right].\end{aligned} \tag{28}$$

This represents the master equation for a two-level atom embedded in a squeezed vacuum reservoir characterized by the parameters N and M.

It is worth mentioning that the atomic operators satisfy the commutation relations $[\hat{\sigma}_\pm, \hat{\sigma}_\mp] = \pm\hat{\sigma}_z$, $[\hat{\sigma}_\pm, \hat{\sigma}_z] = \mp 2\hat{\sigma}_\pm$, and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$. It can also be easily verified that

$$\rho_{aa} = \left\langle \hat{\sigma}_+\hat{\sigma}_- \right\rangle = \frac{\left(\left\langle \hat{\sigma}_z \right\rangle + 1\right)}{2}, \tag{29}$$

$$\rho_{bb} = \left\langle \hat{\sigma}_-\hat{\sigma}_+ \right\rangle \tag{30}$$

represent the probabilities for a two-level atom to be in the upper and lower levels, respectively.

3. Power Spectrum

In nearly all cases, the frequency of a single-mode light is not sharply defined; there is always some spread about the central frequency. We wish here to determine the frequency distribution, usually known as the power spectrum, of the radiation emitted by a two-level atom interacting with a thermal reservoir. We now consider a large group of N identical two-level atoms interacting with some radiation in open space. We note that the number of atoms in the upper and lower levels can be written as

$$N_a(t) = N\rho_{aa}(t) \tag{31}$$

$$N_b(t) = N\rho_{bb}(t). \tag{32}$$

Furthermore, the mean number of atomic excitations can be expressed as

$$\bar{n}(t) = \rho_{aa}(t) = \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t) \right\rangle, \tag{33}$$

with a similar expression for atomic inversion. We can therefore represent the atomic transitions in open space by the atomic operators $\hat{\sigma}_+$ and $\hat{\sigma}_-$, with $\hat{n} = \hat{\sigma}_+\hat{\sigma}_-$ being the photon number operator.

We define the power spectrum of the light emitted by a two-level atom in open space by

$$P(\omega) = \int_{-\infty}^{\infty} d\tau e^{i(\omega-\omega_0)\tau} \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau) \right\rangle_{ss}, \tag{34}$$

where ω_0 is the transition frequency of the atom and ss stands for steady state. It proves to be convenient to rewrite the above integral as

$$\begin{aligned}P(\omega) = &\int_{-\infty}^{0} d\tau e^{i(\omega-\omega_0)\tau} \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau) \right\rangle_{ss} \\ &+ \int_{0}^{\infty} d\tau e^{i(\omega-\omega_0)\tau} \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau) \right\rangle_{ss}.\end{aligned} \tag{35}$$

Assuming that the correlation function at steady state depends only on the time difference τ, one can replace t by $t-\tau$ in the first integral. Thus, upon performing this replacement and then changing the variable of integration from τ to $-\tau$, we have

$$\begin{aligned}P(\omega) = &\int_{0}^{\infty} d\tau e^{i(\omega-\omega_0)\tau} \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau) \right\rangle_{ss} \\ &+ \int_{0}^{\infty} d\tau e^{i(\omega-\omega_0)\tau} \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau) \right\rangle_{ss}.\end{aligned} \tag{36}$$

Since one integral is the complex conjugate of the other, the power spectrum can be put in the form

$$P(\omega) = 2Re\int_{0}^{\infty} d\tau e^{i(\omega-\omega_0)\tau} \left\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau) \right\rangle_{ss}, \tag{37}$$

in which Re denotes the real part.

We next proceed to evaluate the two-time correlation function involved in (37). In accordance with (28), the time evolution of the density operator for a two-level atom coupled to a squeezed vacuum reservoir has the form

$$\begin{aligned}\frac{d}{dt}\hat{\rho} = &\frac{\gamma}{2}(N+1)\left[2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\right] \\ &+ \frac{\gamma}{2}N\left[2\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho} - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\right] \\ &+ \gamma M\left[\hat{\sigma}_-\hat{\rho}\hat{\sigma}_- + \hat{\sigma}_+\hat{\rho}\hat{\sigma}_+\right].\end{aligned} \tag{38}$$

Thus, employing (38) and the cyclic property of the trace operation along with $\hat{\sigma}_-\hat{\sigma}_+\hat{\sigma}_- = \hat{\sigma}_-$, $\hat{\sigma}_+\hat{\sigma}_-\hat{\sigma}_+ = \hat{\sigma}_+$, $\hat{\sigma}_z\hat{\sigma}_\pm = \pm\hat{\sigma}_\pm$, and $\hat{\sigma}_\pm\hat{\sigma}_z = \mp\hat{\sigma}_\pm$, we find

$$\frac{d}{dt}\left\langle \hat{\sigma}_-(t) \right\rangle = -\frac{\gamma}{2}(2N+1)\left\langle \hat{\sigma}_-(t) \right\rangle + \gamma M\left\langle \hat{\sigma}_+(t) \right\rangle, \tag{39}$$

$$\frac{d}{dt}\left\langle \hat{\sigma}_+(t) \right\rangle = -\frac{\gamma}{2}(2N+1)\left\langle \hat{\sigma}_+(t) \right\rangle + \gamma M\left\langle \hat{\sigma}_-(t) \right\rangle, \tag{40}$$

$$\frac{d}{dt}\left\langle \hat{\sigma}_z(t) \right\rangle = -2\gamma N\left\langle \hat{\sigma}_z(t) \right\rangle + 2\gamma\hat{\rho}_{bb}(t). \tag{41}$$

Employing the large-time approximation scheme, one gets from Eq. (40) that

$$\left\langle \hat{\sigma}_+(t) \right\rangle = \left(\frac{2M}{2N+1}\right)\left\langle \hat{\sigma}_-(t) \right\rangle. \tag{42}$$

Hence, the combination of (39) and (42) yields

$$\frac{d}{dt}\left\langle \hat{\sigma}_{-}(t)\right\rangle = -\frac{\gamma\eta}{2}\left\langle \hat{\sigma}_{-}(t)\right\rangle, \tag{43}$$

where

$$\eta = \left[\frac{(2N+1)^2 - 4M^2}{2N+1}\right]. \tag{44}$$

The solution of equation (43) can be written as

$$\left\langle \hat{\sigma}_{-}(t+\tau)\right\rangle = \left\langle \hat{\sigma}_{-}(t)\right\rangle e^{-\gamma\eta\tau/2}, \tag{45}$$

and application of the quantum regression theorem leads to

$$\left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t+\tau)\right\rangle = \left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t)\right\rangle e^{-\gamma\eta\tau/2}. \tag{46}$$

Employing the relation given by (29), one can put Eq. (41) in the form

$$\frac{d}{dt}\rho_{aa}(t) = -\gamma(N+1)\rho_{aa}(t) + \gamma N\rho_{bb}(t). \tag{47}$$

We note that γ is the rate of spontaneous emission and γN represents the rate of absorption or stimulated emission. Employing the identity $\rho_{aa} + \rho_{bb} = 1$, the steady state solution of Eq. (47) is found to be

$$\rho_{aa}(\infty) = \lim_{t\to\infty}\left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t)\right\rangle_{ss} = \frac{N}{2N+1}. \tag{48}$$

Incidentally, we note that for $N \gg 1$, there is nearly equal chance for the atom to be in the upper or lower level.

Now, in view (46) along with (48), the expression for the power spectrum (37) can be put in the form

$$P(\omega) = \frac{2N}{2N+1} Re\int_0^{\infty} d\tau e^{-[(\gamma\eta/2) - i(\omega-\omega_0)]\tau}, \tag{49}$$

so that on carrying out the integration, one readily obtains

$$P(\omega) = \frac{2N}{2N+1}\left[\frac{\gamma\eta/2}{(\gamma\eta/2)^2 + (\omega-\omega_0)^2}\right]. \tag{50}$$

Hence, applying the normalization condition

$$\int_{-\infty}^{\infty} P(\omega)d\omega = 1, \tag{51}$$

and the fact that

$$\int_{-\infty}^{\infty}\frac{d\omega}{a^2 + (\omega-\omega_0)^2} = \frac{\pi}{a}, \tag{52}$$

the normalized power spectrum is found to be

$$P(\omega) = \frac{2N}{2N+1}\left[\frac{\gamma\eta/2\pi}{(\gamma\eta/2)^2 + (\omega-\omega_0)^2}\right]. \tag{53}$$

We immediately notice that the power spectrum is a Lorentzian centered at $\omega = \omega_0$ and with a halfwidth of $\gamma\eta/2$.

Figure 1 shows that the power spectrum of the atomic variables have a single peak centered at $\omega = \omega_0$. It can be observed in the same figure that the height of the power spectrum increases as η decreases. This figure also shows that the width of the spectrum increases with η. This indicates that the squeeze parameter, r, increases the width of the spectrum.

We finally consider the decay of the atoms due to spontaneous emission. We then note that the solution of Eq. (47) with $N = M = 0$ is

$$\rho_{aa}(t) = \rho_{aa}(0)e^{-\gamma t/2}, \tag{54}$$

and if the atom is initially in the upper level

$$\rho_{aa}(t) = e^{-\gamma t/2}. \tag{55}$$

The result given by (55) indicates that the atom in free space decays exponentially with a lifetime of γ. We have found that a two-level atom in a lossless cavity and initially in the upper level makes back and forth oscillations between the two levels even when the cavity mode is initially in a vacuum state. However, the dynamics of such an atom in free space is completely different. In free space, the atom interacts with vacuum modes and this leads to the decay of the atom with a certain lifetime.

It is clearly indicated in Figure 2 as well as in Eq. (55) that the probability for the two-level atom to be on the upper level of the cavity radiation decreases with the decay constant and interaction time. This is related to the fact that as time increases most of the atoms initially in the upper energy level have a chance to decay to the lower energy level and emit radiation. It can be observed in the same way that a significantly intense light can be generated from this system provided that the atoms are allowed to stay in the cavity for a sufficiently long period of time initially present in the cavity, since each atom prefers to stay more often in the lower energy level at steady state as thoroughly discussed elsewhere [20, 30].

4. Second-Order Correlation Function

The second-order correlation function for the light emitted by a two-level atom in open space is expressible as

$$g^{(2)}(\tau) = \lim_{t\to\infty}\frac{\left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{+}(t+\tau)\hat{\sigma}_{-}(t+\tau)\hat{\sigma}_{-}(t)\right\rangle}{\left\langle \sigma\wedge_{+}(t)\sigma\wedge_{-}(t)\right\rangle^2}. \tag{56}$$

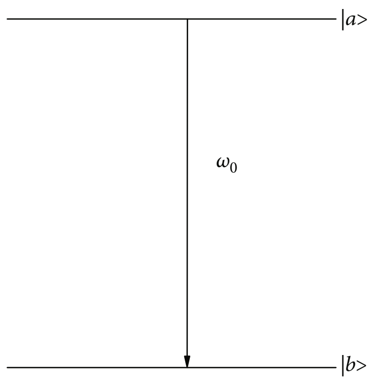

FIGURE 1: Scheme of a two level atom.

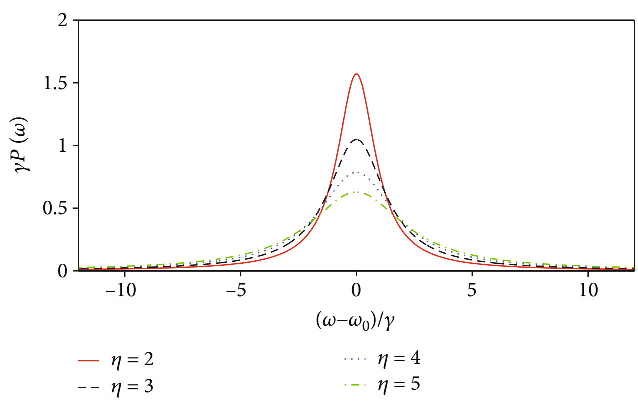

FIGURE 2: Normalized power spectrum for the atomic variables when the two-level atom interacting with a squeezed vacuum [Eq. (53)].

The solution of Eq. (47) is expressible as (Figure 3)

$$\rho_{aa}(t+\tau) = \left[\rho_{aa}(t) - \frac{N}{2N+1}\right] e^{-\gamma\eta\tau/2} + \frac{N}{2N+1}, \qquad (57)$$

so that taking into account (48), we have

$$\left\langle \hat{\sigma}_{+}(t+\tau)\hat{\sigma}_{-}(t+\tau)\right\rangle = \left[\left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t)\right\rangle - \frac{N}{2N+1}\right] \times e^{-\gamma\eta\tau/2} + \frac{N}{2N+1}, \qquad (58)$$

and applying the quantum regression theorem, we get

$$\left\langle \hat{\sigma}_{+}(t+\tau)\hat{\sigma}_{-}(t+\tau)\right\rangle = \frac{N}{2N+1}\left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t)\right\rangle \times \left[1 - e^{-(\gamma\eta\tau/2)}\right]. \qquad (59)$$

Therefore, introducing this result into Eq. (56), there follows

$$g^{(2)}(\tau) = \lim_{t\to\infty} \frac{N}{(2N+1)\left\langle \hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t)\right\rangle}\left[1 - e^{-(\gamma\eta\tau/2)}\right]. \qquad (60)$$

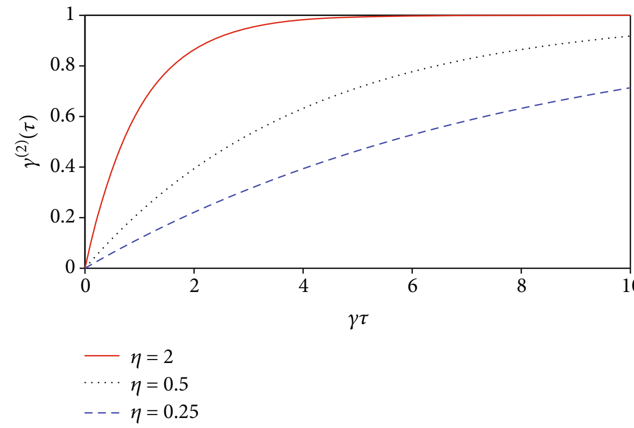

FIGURE 3: Plots of the probability for the two-level atom to be on the upper energy level [Eq. (55)] versus γt.

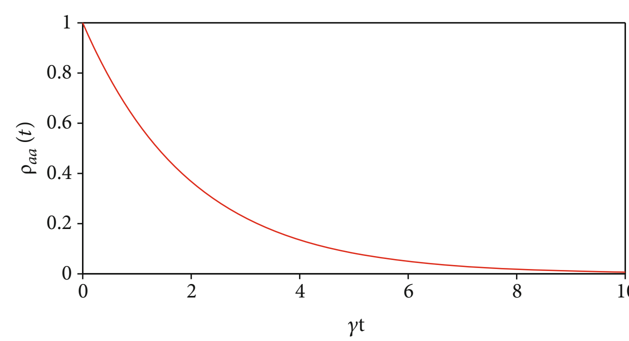

FIGURE 4: Plots of the second-order correlation function versus $\gamma\tau$ for different values of η.

And in view of (48), the expression for the second-order correlation function takes at steady state the form

$$g^{(2)}(\tau) = 1 - e^{-\gamma\eta\tau/2}. \qquad (61)$$

One can observe from the above equation, $g^{(2)}(0) = 0$ and for $\tau > 0$, $g^{(2)}(\tau) > 0$. Therefore, for $\tau > 0$, the second-order correlation function $g^{(2)}(\tau) > g^{(2)}(0)$. It can also be seen from Figure 4 that for relatively small values of τ, the second-order correlation function is less than unity. One can also observe that as η increases, $g^{(2)}(\tau)$ approaches unity at a faster rate. It is also interesting to consider the dynamics of the two-level atom.

5. Conclusion

In this study, we analyzed the quantum properties of a two-level atom embedded in a squeezed vacuum reservoir. Employing the interaction of light with matter, we derived the interaction Hamiltonian and the master equation for the system under consideration. Applying the master equation with the Heisenberg equation, the time evolution of the expectation values of the atomic operators in the bad-cavity limit, is determined. Moreover, using the solutions of the expectation values of the atomic operators, we obtained the power spectrum and the second-order correlation function when the two-level atom interacting with a squeezed vacuum reservoir.

It is found that the effect of squeeze parameter leads to an increase in the width of the power spectrum of the atomic variables of a two-level atom. On the other hand, the effect of squeeze parameter decreases the peak of the power spectrum of the atomic variable. The effect of the squeeze parameter on the second-order correlation function is to enhance its decay rate. Furthermore, in the absence of decay constant and interaction time, it enhances the probability for the atom to be in the upper level.

Data Availability

The article contains theoretical material. This paper has links to articles and textbooks from other researchers and does not use any data.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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