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Research Article

Existence, Nonexistence, and Stability of Solutions for a Delayed Plate Equation with the Logarithmic Source

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In this work, we study a plate equation with time delay in the velocity, frictional damping, and logarithmic source term. Firstly, we obtain the local and global existence of solutions by the logarithmic Sobolev inequality and the Faedo-Galerkin method. Moreover, we prove the stability and nonexistence results by the perturbed energy and potential well methods.

1. Introduction

In this article, we consider a plate equation with frictional damping, delay, and logarithmic terms as follows:

$$\begin{cases} u_{tt} + \Delta^2 u + \alpha u_t(t) + \beta u_t(x, t-\tau) = u \ln |u|^\gamma & \text{for } (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = \dfrac{\partial u(x,t)}{\partial v} = 0 & \text{for } (x,t) \in \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x) & \text{for } x \in \Omega, \\ u_t(x,t) = j_0(x,t) & \text{for } (x,t) \in \Omega \times (-\tau, 0), \end{cases} \tag{1}$$

where $\Omega \subset R^N$, $N \geq 1$, is a bounded domain with smooth boundary $\partial\Omega$. $\tau > 0$ denotes time delay, and α, β, and γ are real numbers that will be specified later. Generally, logarithmic nonlinearity seems to be in supersymmetric field theories and in cosmological inflation. From quantum field theory, that kind of $(u|u|^{p-2} \ln |u|^k)$ logarithmic source term seems to be in nuclear physics, inflation cosmology, geophysics, and optics (see [1, 2]). Time delays often appear in various problems, such as thermal, economic, biological, chemical, and physical phenomena. Recently, partial differential equations have become an active area with time delay (see [3, 4]). In 1986, Datko et al. [5] indicated that, in boundary control, a small delay effect is a source of instability. Generally, a small delay can destabilize a system which is uniformly stable [6]. To stabilize hyperbolic systems with time delay, some control terms will be needed (see [7–9] and references therein).

For the literature review, firstly, we begin with the studies of Bialynicki-Birula and Mycielski [10, 11]. The authors investigated the equation with the logarithmic term as follows:

$$u_{tt} - u_{xx} + u - \varepsilon u \ \ln |u|^2 = 0, \tag{2}$$

where the authors proved that, in any number of dimensions, wave equations including the logarithmic term have localized, stable, soliton-like solutions.

In 1980, Cazenave and Haraux [12] studied the equation as follows:

$$u_{tt} - \Delta u = u \ \ln |u|^k, \tag{3}$$

where the authors in [12] proved the existence and uniqueness of the solutions for equation (3). Gorka [2] obtained the global existence results of solutions for one-dimensional equation (3). Bartkowski and Gŏrka [1] considered the weak solutions and obtained the existence results.

In [13], Hiramatsu et al. studied the equation as follows:

$$u_{tt} - \Delta u + u + u_t + |u^2| u = u \ \ln \ u. \tag{4}$$

In [14], Han established the global existence of solutions for equation (4).

In [15], Al-Gharabli and Messaoudi were concerned with the plate equation with the logarithmic term as follows:

$$u_{tt} + \Delta^2 u + u + h(u_t) = ku \ \ln |u|. \tag{5}$$

They established the existence results by the Galerkin method and obtained the explicit and decay of solutions utilizing the multiplier method for equation (5).

In [16], Liu introduced the plate equation with the logarithmic term as follows:

$$u_{tt} + \Delta^2 u + |u_t|^{m-2} u_t = |u|^{p-2} u \ \log |u|^k. \tag{6}$$

The author proved the local existence by the contraction mapping principle. Also, he studied the global existence and decay results. Moreover, under suitable conditions, the author proved the blow-up results with $E(0) < 0$.

In [17], Messaoudi studied the equation as follows:

$$u_{tt} + \Delta^2 u + |u_t|^{m-2} u_t = |u|^{p-2} u, \tag{7}$$

and obtained the existence results and obtained that, if $m \geq p$, the solution is global and blows up in finite time if $m < p$. Later, Chen and Zhou [18] extended this result. In the presence of the strong damping term $(-\Delta u_t)$, Pişkin and Polat [19] proved the global existence and decay of solutions for equation (7). For more results about plate problems, see [20–22].

In [7], Nicaise and Pignotti studied the equation as follows:

$$u_{tt} - \Delta u + a_0 u_t(x, t) + a u_t(x, t - \tau) = 0, \tag{8}$$

where $a_0, a > 0$. They proved that, under the condition $0 \leq a \leq a_0$, the system is exponentially stable. The authors obtained a sequence of delays that shows the solution is unstable in the case $a \geq a_0$. In the absence of delay, some other authors [23, 24] looked into exponential stability for equation (8). In [9], Xu et al., by using the spectral analysis approach, established the same result similar to [7] for the one space dimension.

In [25], Nicaise et al. studied the wave equation in one space dimension in the presence of time-varying delay. In this article, the authors showed the exponential stability results with the condition

$$a \leq \sqrt{1 - d} a_0, \tag{9}$$

where d is a constant and

$$\tau'(t) \leq d < 1, \quad \forall t > 0. \tag{10}$$

In [26], Kafini and Messaoudi studied wave equations with delay and logarithmic terms as follows:

$$u_{tt} - \Delta u + \mu_1 u_t(x, t) + \mu_2 u_t(x, t - \tau) = |u|^{p-2} u \ \log |u|^k. \tag{11}$$

The authors proved the local existence and blow-up results for equation (11).

In [27], Park considered the equation with delay and logarithmic terms as follows:

$$u_{tt} - \Delta u + \alpha u_t(t) + \beta u_t(x, t - \tau) = u \ \ln |u|^\gamma. \tag{12}$$

The author showed the local and global existence results for equation (12). Also, the author investigated the decay and nonexistence results for equation (12). In recent years, some other authors investigate hyperbolic-type equations with delay terms (see [28–33]).

In this work, we studied the local existence, global existence, nonexistence, and stability results of plate equation (1) with delay and logarithmic terms, motivated by the above works. There is no research, to our best knowledge, related to plate equation (1) with the delay $(\beta u_t(x, t - \tau))$ term and logarithmic $(u \ln |u|^\gamma)$ source term; hence, our work is the generalization of the above studies.

This work consists of five sections in addition to the introduction. Firstly, in Section 2, we recall some assumptions and lemmas. Then, in Section 3, we obtain the local and global existence of solutions. Moreover, in Section 4, we establish the nonexistence results. Finally, in Section 5, we get the stability of solutions.

2. Preliminaries

In this part, we show the norm of X by $\|\cdot\|_X$ for a Banach space X. We give the scalar product in $L^2(\Omega)$ by $(\cdot, \cdot)$. We show $\|\cdot\|_2$ by $\|\cdot\|$, for brevity. Let B_1 be the constant of the embedding inequality

$$\|u\|^2 \leq B_1 \|\Delta u\|^2 \quad \text{for } u \in H_0^2(\Omega). \tag{13}$$

We have the following assumptions related to problem (1):

(H1). The weights of delay and dissipation satisfy

$$0 < |\beta| < \alpha. \tag{14}$$

(H2). The constant γ in (1) satisfies

$$0 < \gamma < \pi e^{(2(N+1))/N}. \tag{15}$$

To get the main result, we have the lemmas as follows.

Lemma 1 (see [34, 35]) (Logarithmic Sobolev inequality). *For any $u \in H_0^1(\Omega)$,*

$$\int_\Omega u^2 \ln |u| dx \leq \frac{1}{2} \|u\|^2 \ln \|u\|^2 + \frac{k^2}{2\pi} \|\nabla u\|^2 - \frac{N}{2}(1 + \ln k)\|u\|^2, \tag{16}$$

where k is a positive real number.

Corollary 2. *For any $u \in H_0^2(\Omega)$,*

$$\int_\Omega u^2 \ln |u| dx \leq \frac{1}{2} \|u\|^2 \ln \|u\|^2 + \frac{k^2}{2\pi} \|\Delta u\|_2^2 - \frac{N}{2}(1 + \ln k)\|u\|^2, \tag{17}$$

where k is a positive real number.

Remark 3. Assume that inequality (17) holds for all $k > 0$, and we choose the constant k that satisfies

$$\rho = \max \left\{ e^{-(N+1)/N}, \mu^{1/N} \sqrt{\frac{\pi}{\gamma}} \right\} < k < \sqrt{\frac{\pi}{\gamma}}, \tag{18}$$

where μ is any real number with

$$0 < \mu < 1. \tag{19}$$

Lemma 4 (see [12]) (Logarithmic Gronwall inequality). *Suppose that $c > 0$ and $l \in L^1(0, T; R^+)$. If a function $f : [0, T] \longrightarrow [1,\infty)$ satisfies*

$$f(t) \leq c\left(1 + \int_0^t l(s) f(s) \ln f(s) ds\right), \quad 0 \leq t \leq T, \tag{20}$$

then

$$f(t) \leq c e^{c \int_0^t l(s) ds}, \quad 0 \leq t \leq T. \tag{21}$$

We define

$$J(v) = \frac{1}{2} \|\Delta v\|^2 - \frac{1}{2} \int_\Omega v^2 \ln |v|^\gamma dx + \frac{\gamma}{4} \|v\|^2, \tag{22}$$

$$I(v) = \|\Delta v\|^2 - \int_\Omega v^2 \ln |v|^\gamma dx, \tag{23}$$

for $v \in H_0^2(\Omega)$; then,

$$J(v) = \frac{1}{2} I(v) + \frac{\gamma}{4} \|v\|^2. \tag{24}$$

Suppose that

$$d = \inf_{v \in H_0^2(\Omega)\setminus\{0\}} \sup_{\lambda \geq 0} J(\lambda v). \tag{25}$$

Then, it satisfies (see, e.g., [36–38])

$$0 < d = \inf_{v \in \mathcal{N}} J(v), \tag{26}$$

where $\mathcal{N}$ is the well-known Nehari manifold, denoted by

$$\mathcal{N} = \left\{ v \in H_0^2(\Omega) \setminus \{0\} \mid I(v) = 0 \right\}. \tag{27}$$

Lemma 5. *I and J are the functions that satisfy*

$$I(\lambda v) = \lambda \frac{\partial J(\lambda v)}{\lambda v} \begin{cases} >0, 0 < \lambda < \lambda^*, \\ = 0, \lambda = \lambda^*, \\ <0, \lambda > \lambda^*, \end{cases} \tag{28}$$

for any $v \in H_0^2(\Omega)$ with $\|v\| \neq 0$, where

$$\lambda^* = \exp\left(\frac{\|\Delta v\|^2 - \int_\Omega v^2 \ln |v|^\gamma dx}{\gamma \|v\|^2} \right). \tag{29}$$

Proof. We obtain, for $\lambda \geq 0$,

$$\begin{aligned} \lambda \frac{\partial}{\partial \lambda} J(\lambda v) &= \lambda \left\{ \lambda \|\Delta v\|^2 - \lambda \int_\Omega v^2 \ln |v|^\gamma dx + \frac{\gamma \lambda}{2} \|v\|^2 \right. \\ &\quad \left. - \lambda \int_\Omega v^2 \ln |\lambda|^\gamma dx - \frac{\gamma \lambda}{2} \int_\Omega v^2 dx \right\} \\ &= \lambda^2 \left(\|\Delta v\|^2 - \int_\Omega v^2 \ln |v|^\gamma dx - \gamma \ln |\lambda| \int_\Omega v^2 dx \right) \\ &= I(\lambda v), \end{aligned} \tag{30}$$

and therefore, we obtain the desired result. □

Remark 6. $J(\lambda v)$ has the absolute maximum value at λ^*, such that

$$\sup_{\lambda \geq 0} J(\lambda v) = J(\lambda^* v) = \exp\left(\frac{2\|\Delta v\|^2 - 2\int_\Omega v^2 \ln |v|^\gamma dx}{\gamma \|v\|^2} \right) \frac{\gamma}{4} \|v\|^2, \tag{31}$$

for $v \in H_0^2(\Omega)$.

Lemma 7. *The potential depth d in (25) satisfies*

$$d \geq \frac{\gamma}{4} e^N \left(\frac{\pi}{\gamma}\right)^{N/2} = E_1. \tag{32}$$

Proof. By Corollary 2, (13), and (18), we have

$$I(v) \geq \left(1 - \frac{k^2\gamma}{2\pi}\right)\|\Delta v\|^2 + \frac{N\gamma}{2}(1+\ln k)\|v\|^2 - \frac{\gamma}{2}\|v\|^2 \ln \|v\|^2 > \frac{N\gamma}{2}(1+\ln k)\|v\|^2 - \frac{\gamma}{2}\|v\|^2 \ln \|v\|^2. \tag{33}$$

Taking the limit $k \longrightarrow \sqrt{\pi/\gamma}$, we obtain

$$I(v) \geq \left\{\frac{N\gamma}{2}\left(1+\ln\sqrt{\frac{\pi}{\gamma}}\right) - \frac{\gamma}{2}\ln\|v\|^2\right\}\|v\|^2. \tag{34}$$

Taking into consideration this and (28), we get

$$0 = I(\lambda^* v) \geq \left\{\frac{N\gamma}{2}\left(1+\ln\sqrt{\frac{\pi}{\gamma}}\right) - \frac{\gamma}{2}\ln\|\lambda^* v\|^2 \|\lambda^* v\|^2\right\}, \tag{35}$$

and therefore,

$$\|\lambda^* v\|^2 \geq e^N\left(\frac{\pi}{\gamma}\right)^{N/2}. \tag{36}$$

Hence, we have by (24) and (31)

$$\sup_{\lambda\geq 0} J(\lambda v) = J(\lambda^* v) = \frac{1}{2}I(\lambda^* v) + \frac{\gamma}{4}\|\lambda^* v\|^2 = \frac{\gamma}{4}\|\lambda^* v\|^2 \geq \frac{\gamma}{4}e^N\left(\frac{\pi}{\gamma}\right)^{N/2}. \tag{37}$$

From the definition of d given in (25), we obtain the result. □

3. Existence

In this part, we have studied the local existence, global existence, nonexistence, and stability results of plate equation (1) with delay and logarithmic terms, motivated by the above works. There is no research, to our best knowledge, related to plate equation (1) with the delay ($\beta u_t(x, t-\tau)$) term and logarithmic ($u \ln |u|^\gamma$) source term; hence, our work is the generalization of the above studies. Firstly, we introduce the new function

$$y(x,\eta,t) = u_t(x, t-\eta\tau) \quad \text{for } (x,\eta,t) \in \Omega\times[0,1]\times(0,\infty). \tag{38}$$

Hence, problem (1) takes the form

$$\begin{cases} u_{tt} + \Delta^2 u + \alpha u_t(x,t) + \beta y(x,1,t) = u \ln|u|^\gamma & \text{for } (x,t)\in\Omega\times(0,\infty), \\ \tau y_t(x,\eta,t) + y_\eta(x,\eta,t) = 0 & \text{for } (x,\eta,t)\in\Omega\times(0,1)\times(0,\infty), \\ u(x,t) = \dfrac{\partial u(x,t)}{\partial \upsilon} = 0 & \text{for } (x,t)\in\partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x) & \text{for } x\in\Omega, \\ y(x,\eta,0) = j_0(x,-\eta\tau) = y_0(x,\eta) & \text{for } (x,\eta)\in\Omega\times(0,1). \end{cases} \tag{39}$$

Definition 8. Assume that $T > 0$. (u, y) is a local solution of problem (39) if it satisfies

$$u \in C([0,T]; H_0^2(\Omega)) \cap C^1([0,T]; L^2(\Omega)) \cap C^2([0,T]; H^{-2}(\Omega)),$$

$$(u_{tt}, v) + (\Delta u, \Delta v) + \alpha(u_t(t), v) + \beta(y(1,t), v) = (u \ln|u|^\gamma, v) \quad \text{for any } v\in H_0^2(\Omega),$$

$$\tau\int_0^1 (y_t(\eta,t), \varphi(\eta))d\eta + \int_0^1 \left(y_\eta(\eta,t), \varphi(\eta)\right)d\eta = 0 \quad \text{for any } \varphi\in L^2(\Omega\times(0,1)),$$

$$u(0) = u_0 \quad \text{in } H_0^2(\Omega),$$

$$u_t(0) = u_1 \quad \text{in } L^2(\Omega),$$

$$y(0) = y_0 \quad \text{in } L^2(\Omega\times(0,1)). \tag{40}$$

3.1. Local Existence. In this part, we establish the local existence results similar to [8, 39].

Theorem 9. *Suppose that (H1) and (H2) are satisfied. Then, for the initial data $u_0 \in H_0^2(\Omega)$, $u_1 \in L^2(\Omega)$, and $y_0 \in L^2(\Omega\times(0,1))$, there exists a local solution (u, y) for problem (39).*

Proof. Let $\{v_i\}_{i\in N}$ be the orthogonal basis of $H_0^2(\Omega)$ that is orthonormal in $L^2(\Omega)$. Define $\varphi_i(x,0) = v_i(x)$, and we extend $\varphi_i(x,0)$ by $\varphi_i(x,\eta)$ over $L^2(\Omega\times(0,1))$. We denote $V_n = \text{span}\{v_1, v_2, \cdots, v_n\}$ and $W_n = \text{span}\{\varphi_1, \varphi_2, \cdots, \varphi_n\}$ for $n \geq 1$. We consider the Faedo-Galerkin approximation solution $(u^n, y^n) \in V_n \times W_n$ of the form

$$u^n = \sum_{i=1}^n h_i^n(t) v_i(x),$$
$$y^n(x,\eta,t) = \sum_{i=1}^n g_i^n(t)\varphi_i(x,\eta), \quad n = 1, 2, \cdots, \tag{41}$$

solving the approximate system

$$(u_{tt}^n, v) + (\Delta u^n, \Delta v) + \alpha(u_t^n(t), v) + \beta(y^n(1,t), v) = \int_\Omega u^n \ln|u^n|^\gamma v dx \quad \text{for } v\in V_n, \tag{42}$$

$$\tau\int_0^1 (y_t^n(\eta,t),\varphi(\eta))d\eta + \int_0^1 \left(y_\eta^n(\eta,t),\varphi(\eta)\right)d\eta = 0 \quad \text{for } \varphi \in W_n, \tag{43}$$

$$\begin{aligned} u^n(0) &= u_0^n, \\ u_t^n(0) &= u_1^n, \\ y^n(0) &= y_0^n, \end{aligned} \tag{44}$$

where

$$\begin{aligned} u_0^n &\longrightarrow u_0 \quad \text{in } H_0^2(\Omega), \\ u_1^n &\longrightarrow u_1 \quad \text{in } L^2(\Omega), \\ y_0^n &\longrightarrow y_0 \quad \text{in } L^2(\Omega\times(0,1)). \end{aligned} \tag{45}$$

Since problem (42)–(44) is a normal system of ordinary differential equations, there exists a solution (u^n, y^n) on the interval $[0, t_n)$, $t_n \in (0, T]$. The extension of that solution to the $[0, T)$ is a consequence of the estimate below.

By replacing v by $u_t^n(t)$ in (42) and by using the relation

$$\int_\Omega u^n \ln |u^n|^\gamma u_t^n dx = \frac{d}{dt}\left\{\frac{1}{2}\int_\Omega (u^n)^2 \ln |u^n|^\gamma dx - \frac{\gamma}{4}\|u^n\|^2\right\}, \tag{46}$$

we have

$$\begin{aligned} &\frac{d}{dt}\left\{\frac{1}{2}\|u_t^n\|^2 + \frac{1}{2}\|\Delta u^n\|^2 + \frac{\gamma}{4}\|u^n\|^2 - \frac{1}{2}\int_\Omega (u^n)^2 \ln |u^n|^\gamma dx\right\} \\ &\quad = -\alpha\|u_t^n(t)\|^2 - \beta(y^n(1,t), u_t^n(t)). \end{aligned} \tag{47}$$

By replacing φ by $\omega y^n(\eta, t)$ in (43), we see that

$$\frac{\omega\tau}{2}\frac{d}{dt}\int_\Omega\int_0^1 (y^n(x,\eta,t))^2 d\eta dx = -\frac{\omega}{2}\|y^n(1,t)\|^2 + \frac{\omega}{2}\|y^n(0,t)\|^2. \tag{48}$$

Summing (47) and (48), we obtain

$$\frac{d}{dt}E^n(t) = -\alpha\|u_t^n\|^2 - \beta(y^n(1,t), u_t^n(t)) - \frac{\omega}{2}\|y^n(1,t)\|^2 + \frac{\omega}{2}\|y^n(0,t)\|^2, \tag{49}$$

where

$$\begin{aligned} E^n(t) = &\frac{1}{2}\|u_t^n\|^2 + \frac{1}{2}\|\Delta u^n\|^2 + \frac{\gamma}{4}\|u^n\|^2 - \frac{1}{2}\int_\Omega (u^n)^2 \ln |u^n|^\gamma dx \\ &+ \frac{\omega\tau}{2}\|y^n\|^2_{L^2(\Omega\times(0,1))}, \end{aligned} \tag{50}$$

where

$$|\beta| < \omega < 2\alpha - |\beta|. \tag{51}$$

Utilizing Young's inequality and the fact that $y^n(x,0,t) = u_t^n(x,t)$, we obtain

$$\frac{d}{dt}E^n(t) \le -\left(\alpha - \frac{|\beta|}{2} - \frac{\omega}{2}\right)\|u_t^n\|^2 - \left(\frac{\omega}{2} - \frac{|\beta|}{2}\right)\|y^n(1,t)\|^2 \le 0, \tag{52}$$

$$E^n(t) + C_1\int_0^t \|u_t^n(s)\|^2 ds + C_2\int_0^t \|y^n(1,s)\|^2 ds \le E^n(0), \tag{53}$$

where

$$\begin{aligned} C_1 &= \alpha - \frac{|\beta|}{2} - \frac{\omega}{2} > 0, \\ C_2 &= \frac{\omega}{2} - \frac{|\beta|}{2} > 0. \end{aligned} \tag{54}$$

Taking into consideration this and Corollary 2, we have

$$\begin{aligned} &\|u_t^n\|^2 + \left(1 - \frac{\gamma k^2}{2\pi}\right)\|\Delta u^n\|^2 + \frac{\gamma}{2}(1 + N(1 + \ln k))\|u^n\|^2 \\ &\quad + 2C_1\int_0^t \|u_t^n(s)\|^2 ds + 2C_2\int_0^t \|y^n(1,s)\|^2 ds + \omega\tau\|y^n\|^2_{L^2(\Omega\times(0,1))} \\ &\quad \le 2E^n(0) + \frac{\gamma}{2}\|u^n\|^2 \ln \|u^n\|^2. \end{aligned} \tag{55}$$

By using (18), we obtain

$$\begin{aligned} &1 - \frac{\gamma k^2}{2\pi} > 0, \\ &\frac{\gamma}{2}(1 + N(1 + \ln k)) > 0, \end{aligned} \tag{56}$$

and therefore,

$$\begin{aligned} &\|u_t^n\|^2 + \|\Delta u^n\|^2 + \|u^n\|^2 + \int_0^t \|u_t^n(s)\|^2 ds + \int_0^t \|y^n(1,s)\|^2 ds \\ &\quad + \|y^n\|^2_{L^2(\Omega\times(0,1))} \le c_1\left(1 + \|u^n\|^2 \ln \|u^n\|^2\right), \end{aligned} \tag{57}$$

where the sequel c_j, $j = 1, 2, \cdots$, shows a positive constant. Also, we know that

$$u^n(x,t) = u^n(x,0) + \int_0^t u_t^n(x,s)ds. \tag{58}$$

Utilizing Cauchy-Schwarz's inequality and (57), we obtain

$$\begin{aligned}\|u^n(t)\|^2 &= 2\|u^n(0)\|^2 + 2T\int_0^t \|u_t^n(s)\|^2 ds \\ &\le 2\|u^n(0)\|^2 + 2T\int_0^t c_1\left(1 + \|u^n(s)\|^2 \ln \|u^n(s)\|^2\right) ds \\ &\le c_2\left(1 + \int_0^t \|u^n(s)\|^2 \ln \|u^n(s)\|^2 ds\right).\end{aligned} \tag{59}$$

From Lemma 4, we arrive at

$$\|u^n(t)\|^2 \le c_3 e^{c_4 T}. \tag{60}$$

$f(s) = s \ln s$ is the function which is continuous on $(0, \infty)$, $\lim_{s\longrightarrow 0^+} f(s) = 0$, $\lim_{s\longrightarrow +\infty} f(s) = +\infty$, and f decreases on $(0, e^{-1})$ and increases on $(e^{-1}, +\infty)$; hence, we get by (57) and (60)

$$\begin{aligned}&\|u_t^n\|^2 + \|\Delta u^n\|^2 + \|u^n\|^2 + \int_0^t \|u_t^n(s)\|^2 ds + \int_0^t \|y^n(1,s)\|^2 ds \\ &\quad + \|y^n\|^2_{L^2(\Omega\times(0,1))} \le c_5.\end{aligned} \tag{61}$$

Hence, there exists a subsequence of $\{(u^n, y^n)\}$, which we still denote $\{(u^n, y^n)\}$, such that

$$\begin{aligned}&u^n \longrightarrow u \quad \text{weakly star in } L^\infty\left(0, T; H_0^2(\Omega)\right), \\ &u_t^n \longrightarrow u_t \quad \text{weakly star in } L^\infty\left(0, T; L^2(\Omega)\right), \\ &y^n \longrightarrow y \quad \text{weakly star in } L^\infty\left(0, T; L^2(\Omega\times(0,1))\right), \\ &y^n(1) \longrightarrow y(1) \quad \text{weakly in } L^2\left(0, T; L^2(\Omega)\right).\end{aligned} \tag{62}$$

Utilizing the Aubin-Lions compactness theorem, we conclude that

$$\begin{aligned}&u^n \longrightarrow u \quad \text{strongly in } L^2\left(0, T; L^2(\Omega)\right), \\ &u^n \longrightarrow u \quad \text{a.e.in } \Omega\times(0, T).\end{aligned} \tag{63}$$

The function $s \longrightarrow s \ln |s|^\gamma$ is continuous on R; hence,

$$u^n \ln |u^n|^\gamma \longrightarrow u \ln |u|^\gamma \quad \text{a.e.in } \Omega\times(0, T). \tag{64}$$

Let

$$\begin{aligned}\Omega_1 &= \{x\in\Omega \mid |u^n| < 1\}, \\ \Omega_2 &= \{x\in\Omega \mid |u^n| \ge 1\}.\end{aligned} \tag{65}$$

Thus, we obtain

$$\begin{aligned}\int_\Omega (u^n \ln |u^n|^\gamma)^2 dx &= \gamma^2\left\{\int_{\Omega_1} (u^n \ln |u^n|)^2 dx + \int_{\Omega_2} (u^n \ln |u^n|)^2 dx\right\} \\ &\le \gamma^2\left\{e^{-2}|\Omega_1| + e^{-2}\left(\frac{2}{q-2}\right)^2 \int_{\Omega_2} (u^n)^q dx\right\} \quad \text{for any } q > 2,\end{aligned} \tag{66}$$

where we used

$$\begin{aligned}&|s \ln s| \le \frac{1}{e} \quad \text{for } 0 < s < 1, \\ &s^{-\kappa} \ln s \le \frac{1}{e\kappa} \quad \text{for } s \ge 1 \text{ and } \kappa > 0.\end{aligned} \tag{67}$$

By (57) and (66), we conclude that

$$\int_\Omega (u^n \ln |u^n|^\gamma)^2 dx \le \gamma^2\left\{e^{-2}|\Omega_1| + e^{-2}\left(\frac{2}{q-2}\right)^2 B_2^q \|\Delta u^n\|^q\right\} \le c_6, \tag{68}$$

where B_2 is the Sobolev imbedding constant of

$$H_0^2(\Omega) \subset L^q(\Omega) \quad \text{for } q > 2, \text{ if } N = 1, 2, 3, 4; 2 < q < \frac{2N}{N-4}, \text{ if } N \ge 5. \tag{69}$$

Therefore, we get from (68)

$$u^n \ln |u^n|^\gamma \quad \text{which is uniformly bounded in } L^\infty\left(0, T; L^2(\Omega)\right). \tag{70}$$

From the Lebesgue bounded convergence theorem, (64), and (70), we arrive at

$$u^n \ln |u^n|^\gamma \longrightarrow u \ln |u|^\gamma \quad \text{strongly in } L^2\left(0, T; L^2(\Omega)\right). \tag{71}$$

We pass the limit $m \longrightarrow \infty$ in (42) and (43). The remainder of the proof is standard and similar to [39, 40]. □

3.2. Global Existence. In this part, we obtain the global existence results for problem (39). For this goal, we define the energy functional of problem (39):

$$\begin{aligned}E(t) &= \frac{1}{2}\|u_t\|^2 + \frac{1}{2}\|\Delta u\|^2 + \frac{\gamma}{4}\|u\|^2 - \frac{1}{2}\int_\Omega u^2 \ln |u|^\gamma dx \\ &\quad + \frac{\omega\tau}{2}\|y\|^2_{L^2(\Omega\times(0,1))},\end{aligned} \tag{72}$$

where ω is the positive constant given in (51). We see that

$$\begin{aligned}E(t) &= \frac{1}{2}\|u_t\|^2 + J(u(t)) + \frac{\omega\tau}{2}\|y\|^2_{L^2(\Omega\times(0,1))} = \frac{1}{2}\|u_t\|^2 \\ &\quad + \frac{1}{2}I(u(t)) + \frac{\gamma}{4}\|u\|^2 + \frac{\omega\tau}{2}\|y\|^2_{L^2(\Omega\times(0,1))}.\end{aligned} \tag{73}$$

By the same arguments similar to (52), we infer that

$$\frac{d}{dt}E(t) \leq -C_1\|u_t\|^2 - C_2\|y(1,t)\|^2 \leq 0, \tag{74}$$

where C_1 and C_2, given in (54), are positive constants.

Lemma 10. *Suppose that (H1) and (H2) are satisfied. If $E(0) < d$ and $I(u_0) > 0$, then the solution u of problem (1) satisfies*

$$I(u(t)) > 0 \quad \text{for } t \in [0, T), \tag{75}$$

where T is the maximal existence time of the solutions.

Proof. We know that $I(u_0) > 0$ and u is continuous on $[0, T)$; hence, we have

$$I(u(t)) > 0 \quad \text{for some interval } [0, t_1) \in [0, T). \tag{76}$$

Let t_0 be the maximum of t_1 satisfying (76). Assume that $t_0 < T$; then, $I(u(t_0)) = 0$, that is,

$$u(t_0) \in \mathcal{N}. \tag{77}$$

Therefore, we obtain by (26)

$$J(u(t_0)) \geq \inf_{v\in\mathcal{N}} J(v) = d. \tag{78}$$

We see that this is in contradiction to the relation as follows:

$$J(u(t_0)) \leq E(t_0) \leq E(0) < d. \tag{79}$$

By (74) and Lemma 10, we see that $E(t)$ is a nonincreasing function. □

Theorem 11. *The solution u is global, under the conditions of Lemma 10.*

Proof. It suffices to show that $\|u_t\|^2 + \|\Delta u\|^2$ is bounded independent of t. By Lemma 10, (73), and (74), we get

$$\|u_t\|^2 \leq \|u_t\|^2 + I(u(t)) \leq 2E(t) \leq 2E(0) < 2d. \tag{80}$$

In a similar way, we get

$$\|u\|^2 < \|u\|^2 + \frac{2}{\gamma}I(u(t)) = \frac{4}{\gamma}J(u(t)) \leq \frac{4}{\gamma}E(t) \leq \frac{4}{\gamma}E(0) < \frac{4d}{\gamma}. \tag{81}$$

By Corollary 2 and (23), we conclude that

$$\begin{aligned}\|\Delta u\|^2 = I(u(t)) + \gamma\int_\Omega u^2 \ln|u|dx \leq 2E(t) + \frac{\gamma}{2}\|u\|^2 \ln\|u\|^2 \\ + \frac{k^2\gamma}{2\pi}\|\Delta u\|^2 - \frac{N\gamma}{2}(1+\ln k)\|u\|^2.\end{aligned} \tag{82}$$

By taking the limit $k \longrightarrow \rho^+$ in this inequality and from (81), we obtain

$$\begin{aligned}\left(1 - \frac{\rho^2\gamma}{2\pi}\right)\|\Delta u\|^2 &\leq 2E(t) + \frac{\gamma}{2}\left(\ln\|u\|^2 - N(1+\ln\rho)\right)\|u\|^2 \\ &< 2d + \frac{\gamma}{2}\left(\ln\left(\frac{4d}{\gamma}\right)\right) - N(1+\ln\rho)\|u\|^2 \\ &= 2d + \frac{\gamma}{2}\left\{\ln\left(\frac{4d}{\gamma}e^{-N}\rho^{-N}\right)\right\}\|u\|^2.\end{aligned} \tag{83}$$

By Lemma 7 and (18), we get

$$\begin{aligned}\ln\left(\frac{4d}{\gamma}e^{-N}\rho^{-N}\right) &\geq \ln\left(\left(\frac{\pi}{\gamma}\right)^{N/2}\rho^{-N}\right) \\ &= \ln\left(\left(\sqrt{\frac{\pi}{\gamma}}\rho^{-1}\right)^N\right) \ln 1 = 0.\end{aligned} \tag{84}$$

Therefore, we see by (81) and (83) that

$$\left(1 - \frac{\rho^2\gamma}{2\pi}\right)\|\Delta u\|^2 \leq 2d + 2d\ \ln\left(\frac{4d}{\gamma}e^{-N}\rho^{-N}\right). \tag{85}$$

Hence, we conclude that

$$\|\Delta u\|^2 < 2d\left(1 - \frac{\rho^2\gamma}{2\pi}\right)^{-1}\left(1 + \ln\left(\frac{4d}{\gamma}e^{-N}\rho^{-N}\right)\right). \tag{86}$$

Therefore, we complete the proof by (80) and (86). □

4. Nonexistence

In this part, similar to [41–43], we get the nonexistence results for problem (1). Firstly, we need the lemma as follows.

Lemma 12. *Assume that (H1) and (H2) are satisfied. If $E(0) < E_1$ and $I(u_0) < 0$, then the solution u of problem (1) satisfies*

$$I(u(t)) < 0 \quad \text{for } t \in [0, T), \tag{87}$$

$$\|u(t)\|^2 > \frac{4E_1}{\gamma} \quad \text{for } t \in [0, T), \tag{88}$$

where T is the maximal existence time of the solutions.

Proof. We know that $I(u_0) < 0$ and u is continuous on $[0, T)$; hence, we have

$$I(u(t)) < 0 \quad \text{for some interval } [0, t_1) \subset [0, T). \tag{89}$$

Let t_0 be the maximal time satisfying (89) and assume that $t_0 < T$; then, $I(u_0) = 0$, such that

$$u(t_0) \in \mathcal{N}. \tag{90}$$

Therefore, we obtain

$$d \le J(u(t_0)) = \frac{1}{2} I(u(t_0)) + \frac{\gamma}{4} \|u(t_0)\|^2 \le E(u(t_0)) \le E(0) < E_1. \tag{91}$$

This is in contradiction to Lemma 7. Thus, (87) is proved. By Lemma 7, (31), and (87), we conclude that

$$E_1 \le d \le J(\lambda^* u(t)) = \exp\left(\frac{2\|\Delta u\|^2 - 2\int_\Omega u^2 \ln |u|^\gamma dx}{\gamma \|u\|^2}\right) \frac{\gamma}{4} \|u\|^2 < \frac{\gamma}{4} \|u\|^2. \tag{92}$$

Therefore, the proof is completed. □

Theorem 13. *Suppose that (H1) and (H2) are satisfied. Let $E(0) < \zeta E_1$, where $0 < \zeta < 1$, and $I(u_0) < 0$. Then, the solution of problem (1) blows up at infinity.*

Proof. Firstly, we set

$$F(t) = \zeta E_1 - E(t). \tag{93}$$

By (74), we obtain

$$F'(t) = -E'(t) \ge C_1 \|u_t\|^2 + C_2 \|y(1,t)\|^2 \ge 0. \tag{94}$$

Utilizing (72), (88), and (94), we see that

$$0 < F(0) \le F(t) \le \zeta E_1 + \frac{1}{2}\int_\Omega u^2 \ln |u|^\gamma dx < \frac{\gamma}{4}\|u\|^2 + \frac{1}{2}\int_\Omega u^2 \ln |u|^\gamma dx. \tag{95}$$

We define

$$G(t) = F(t) + \varepsilon(u, u_t) + \frac{\varepsilon\alpha}{2}\|u\|^2. \tag{96}$$

By (39) and (72), we get

$$\begin{aligned} G'(t) &= F'(t) + \varepsilon\|u_t\|^2 - \varepsilon\|\Delta u\|^2 - \varepsilon\beta(u, y(1,t)) \\ &\quad + \varepsilon\int_\Omega u^2 \ln |u|^\gamma dx = F'(t) + 2\varepsilon\|u_t\|^2 - \varepsilon\beta(u, y(1,t)) \\ &\quad - 2\varepsilon E(t) + \frac{\varepsilon\gamma}{2}\|u\|^2 + \omega\tau\|y\|^2_{L^2(\Omega\times(0,1))}. \end{aligned} \tag{97}$$

Utilizing Young's inequality and (94), we obtain

$$\beta(u, y(1,t)) \le |\beta|\left(\delta\|u\|^2 + \frac{1}{4\delta}\|y(1,t)\|^2\right) \le \delta|\beta|\|u\|^2 + \frac{|\beta|}{4\delta C_2}F'(t). \tag{98}$$

By adapting this to (97) and from (88) and (93), we have

$$\begin{aligned} G'(t) &\ge \left(1 - \frac{\varepsilon|\beta|}{4\delta C_2}\right)F'(t) + 2\varepsilon\|u_t\|^2 + \left(\frac{\varepsilon\gamma}{2} - \varepsilon|\beta|\delta\right)\|u\|^2 \\ &\quad + 2\varepsilon F(t) - 2\varepsilon\zeta E_1 + \omega\tau\|y\|^2_{L^2(\Omega\times(0,1))} \\ &\ge \left(1 - \frac{\varepsilon|\beta|}{4\delta C_2}\right)F'(t) + 2\varepsilon\|u_t\|^2 + \varepsilon\left((1-\zeta)\frac{\gamma}{2} - |\beta|\delta\right)\|u\|^2 \\ &\quad + 2\varepsilon F(t) + \omega\tau\|y\|^2_{L^2(\Omega\times(0,1))}. \end{aligned} \tag{99}$$

Firstly, fix $\delta > 0$ such that $(1-\zeta)(\gamma/2) - |\beta|\delta > 0$ and then choose $\varepsilon > 0$ small enough so that $1 - (\varepsilon|\beta|/4\delta C_2) > 0$. Then, by (94), we get

$$G'(t) \ge c_8\left(F(t) + \|u_t\|^2 + \|u\|^2\right) \ge 0. \tag{100}$$

Also, we conclude that

$$G(t) \le c_9\left(F(t) + \|u_t\|^2 + \|u\|^2\right). \tag{101}$$

Taking $\varepsilon > 0$ small enough again, we obtain

$$G(0) = F(0) + \varepsilon(u_0, u_1) + \frac{\varepsilon\alpha}{2}\|u_0\|^2 > 0. \tag{102}$$

By (100) and (102), we get

$$G(t) \ge G(0) > 0. \tag{103}$$

Utilizing (100) and (101), we see that

$$G'(t) \ge c_{10} G(t), \tag{104}$$

and therefore,

$$G(t) \ge e^{c_{10} t} G(0) > 0. \tag{105}$$

Therefore, $G(t)$ blows up at infinity. Consequently, the proof is completed. □

5. Stability

In this part, we obtain the stability of global solutions. Firstly, we define the perturbed energy by

$$\Psi(t) = E(t) + \varepsilon\Phi(t) + \varepsilon\Xi(t), \tag{106}$$

where $\varepsilon > 0$, $\Phi(t) = (u_t, u)$, and $\Xi(t) = \int_\Omega\int_0^1 e^{-\tau\eta} y^2(x,\eta,t) d\eta\, dx$.

Lemma 14. *Under the conditions of Lemma 10, for $C_3, C_4 > 0$, we obtain*

$$C_3 E(t) \le \Psi(t) \le C_4 E(t). \tag{107}$$

Proof. Utilizing Lemma 10 and Young's inequality, we have

$$\begin{aligned} |\Phi(t)+\Xi(t)| &\le \frac{1}{2}\|u_t\|^2+\frac{1}{2}\|u\|^2+\|y\|^2_{L^2(\Omega\times(0,1))} \\ &\le \frac{1}{2}\|u_t\|^2+\frac{2}{\gamma}\left(\frac{\gamma}{4}\|u\|^2+\frac{1}{2}I(u(t))\right) \\ &\quad+\|y\|^2_{L^2(\Omega\times(0,1))}=\frac{1}{2}\|u_t\|^2+\frac{2}{\gamma}J(u(t)) \\ &\quad+\|y\|^2_{L^2(\Omega\times(0,1))}\le c_7E(t). \end{aligned} \tag{108}$$

Taking $\varepsilon>0$ small enough, we complete the proof. □

Theorem 15. *Assume that (H1) and (H2) are satisfied. Suppose that $E(0)<E_1$ and $I(u_0)>0$. Hence, for $C_0, C_5>0$, we obtain*

$$0<E(t)\le C_0e^{-C_5t}\quad \text{for } t\ge 0. \tag{109}$$

Proof. From (39) and Young's inequality, we get

$$\begin{aligned} \Phi'(t) &= \|u_t\|^2-\|\Delta u\|^2-\alpha(u_t(t),u(t))-\beta(y(1,t),u(t)) \\ &\quad+\int_\Omega u^2\ln|u|^\gamma dx\le\|u_t\|^2-\frac{1}{2}\|\Delta u\|^2+\alpha^2B_1\|u_t(t)\|^2 \\ &\quad+\beta^2B_1\|y(1,t)\|^2+\int_\Omega u^2\ln|u|^\gamma dx. \end{aligned} \tag{110}$$

By using the second equation of (39) and the integration by parts, we obtain

$$\begin{aligned} \Xi'(t) &= -\frac{2}{\tau}\int_\Omega\int_0^1 e^{-\tau\eta}y(x,\eta,t)y_\eta(x,\eta,t)d\eta dx \\ &= -\frac{1}{\tau}\int_\Omega\int_0^1 e^{-\tau\eta}\frac{\partial}{\partial\eta}y^2(x,\eta,t)d\eta dx \\ &= -\frac{e^{-\tau}}{\tau}\|y(1,t)\|^2+\frac{1}{\tau}\|y(0,t)\|^2-\int_\Omega\int_0^1 e^{-\tau\eta}y^2(x,\eta,t)d\eta dx \\ &\le \frac{1}{\tau}\|u_t\|^2-e^{-\tau}\int_\Omega\int_0^1 y^2(x,\eta,t)d\eta dx. \end{aligned} \tag{111}$$

Summing these and (74), we obtain

$$\begin{aligned} \Psi'(t) &\le -\left(C_1-\varepsilon-\varepsilon\alpha^2B_1-\frac{\varepsilon}{\tau}\right)\|u_t\|^2-\frac{\varepsilon}{2}\|\Delta u\|^2 \\ &\quad-\left(C_2-\varepsilon\beta^2B_1\right)\|y(1,t)\|^2+\varepsilon\int_\Omega u^2\ln|u|^\gamma dx \\ &\quad-\varepsilon e^{-\tau}\|y\|^2_{L^2(\Omega\times(0,1))}. \end{aligned} \tag{112}$$

Adding and subtracting $\xi E(t)$ with $0<\xi<2\varepsilon$, we get

$$\begin{aligned} \Psi'(t) &\le -\xi E(t)-\left(C_1-\varepsilon-\varepsilon\alpha^2B_1-\frac{\varepsilon}{\tau}-\frac{\xi}{2}\right)\|u_t\|^2 \\ &\quad-\left(\frac{\varepsilon}{2}-\frac{\xi}{2}-\frac{\xi\gamma B_1}{4}\right)\|\Delta u\|^2-\left(C_2-\varepsilon\beta^2B_1\right)\|y(1,t)\|^2 \\ &\quad+\left(\varepsilon-\frac{\xi}{2}\right)\int_\Omega u^2\ln|u|^\gamma dx-\left(\varepsilon e^{-\tau}-\frac{\xi\omega\tau}{2}\right)\|y\|^2_{L^2(\Omega\times(0,1))}. \end{aligned} \tag{113}$$

Utilizing the logarithmic Sobolev inequality, we have

$$\begin{aligned} \Psi'(t) &\le -\xi E(t)-\left(C_1-\varepsilon-\varepsilon\alpha^2B_1-\frac{\varepsilon}{\tau}-\frac{\xi}{2}\right)\|u_t\|^2 \\ &\quad-\left\{\varepsilon\left(\frac{1}{2}-\frac{\gamma k^2}{2\pi}\right)-\frac{\xi}{2}\left(1-\frac{\gamma k^2}{2\pi}\right)-\frac{\xi\gamma B_1}{4}\right\}\|\Delta u\|^2 \\ &\quad+\frac{\gamma}{2}\left(\varepsilon-\frac{\xi}{2}\right)\left\{\ln\|u\|^2-N(1+\ln k)\right\}\|u\|^2 \\ &\quad-\left(C_2-\varepsilon\beta^2B_1\right)\|y(1,t)\|^2-\left(\varepsilon e^{-\tau}-\frac{\xi\omega\tau}{2}\right)\|y\|^2_{L^2(\Omega\times(0,1))}. \end{aligned} \tag{114}$$

Now, choose $\varepsilon>0$ small enough, such that

$$\begin{aligned} &C_1-\varepsilon-\varepsilon\alpha^2B_1-\frac{\varepsilon}{\tau}>0, \\ &C_2-\varepsilon\beta^2B_1>0. \end{aligned} \tag{115}$$

By taking $\xi>0$ sufficiently small and noting that $(1/2)-(\gamma k^2/2\pi)>0$ (see (18)), we infer that

$$\Psi'(t)\le-\xi E(t)+\frac{\gamma}{2}\left(\varepsilon-\frac{\xi}{2}\right)\left\{\ln\|u\|^2-N(1+\ln k)\right\}\|u\|^2, \tag{116}$$

where $0<E(0)<E_1$; therefore, there exists $0<\mu<1$, that is, $E(0)=\mu E_1$. Therefore, we obtain by (81)

$$\begin{aligned} \ln\|u\|^2 &< \ln\left(\frac{4}{\gamma}E(t)\right)\le\ln\left(\frac{4}{\gamma}E(0)\right)=\ln\left(\frac{4\mu E_1}{\gamma}\right) \\ &= \ln\left(\mu e^N\left(\frac{\pi}{\gamma}\right)^{N/2}\right). \end{aligned} \tag{117}$$

Hence, by (18), we arrive at

$$\begin{aligned} \ln\|u\|^2-N(1+\ln k) &\le \ln\left(\mu e^N\left(\frac{\pi}{\gamma}\right)^{N/2}\right)-N(1+\ln k) \\ &= N\ln\left(\mu^{1/N}\sqrt{\frac{\pi}{\gamma}}k^{-1}\right)<N\ln 1=0. \end{aligned} \tag{118}$$

Substituting this into (116), we arrive at

$$\Psi'(t) \leq -\xi E(t). \quad (119)$$

As a result, from Lemma 14, we completed the proof. □

6. Conclusions

Recently, there have been many published works related to wave equations with time delay. There were no local existence, global existence, nonexistence, and stability results of the plate equation with delay and logarithmic source terms, to the best of our knowledge. Firstly, we have obtained the local and global existence results. Then, we have obtained the nonexistence of solutions. Finally, we have proved stability results under sufficient conditions.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no competing interests.

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